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Reciprocal Learning in Mathematics Education: An Interactive Study Between Two Canadian and Chinese Elementary Schools

L'apprentissage réciproque dans l'enseignement mathématique : une étude interactive entre deux écoles élémentaires canadiennes et chinoises

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Abstract

In this study, the researchers go beyond the back-and-forth debates on the East-West educational paradigms that often arise from comparative studies, and take a reciprocal learning approach to explore the commonalities and differences in mathematics education between two Canadian and Chinese elementary schools. Research data were collected through direct and indirect interactions between the pair of research schools, including Skype meetings; formal and informal conversations with teachers and administrators; and the sharing/exchange of documents, texts, teaching materials, and resources. Results show that there is a common emphasis on some thematic issues in the teaching and learning of mathematics including the use of manipulatives, multiple solutions to mathematical problems, and parental involvement, but also some differences between the two schools in teachers' strategies for teaching problem solving, students' learning tendencies and schools' supports for special needs students. The researchers conclude that the dichotomies of the East-West educational paradigms need to be further and more deeply re-examined.

Résumé

Dans cette étude, les chercheurs vont au-delà des débats incessants sur le paradigme éducationnel Est-Ouest qui proviennent souvent d'études comparatives et prennent une approche d'apprentissage réciproque pour explorer les points communs et les différences dans l'éducation des mathématiques entre deux écoles élémentaires canadiennes et chinoises. Les données de la recherche ont été recueillies par le biais d'interactions directes et indirectes entre les deux écoles de notre échantillon, incluant des rencontres Skype; des conversations formelles et informelles avec les enseignants et l'administration ainsi que le partage/échange de documents, de textes, de matériel d'enseignement et de ressources. Les résultats montrent qu'il y a une emphase commune sur certains enjeux thématiques concernant l'enseignement et l'apprentissage des mathématiques, incluant l'usage de manipulations, de solutions multiples pour les problèmes mathématiques et l'implication parentale, mais aussi quelques différences entre les deux écoles concernant les stratégies d'enseignement de la résolution de problème, le style d'apprentissage des élèves et l'appui de l'école pour les élèves ayant des besoins spéciaux. Les chercheurs concluent que la dichotomie du paradigme éducationnel Est-West a besoin d'être réexaminée davantage et plus en profondeur.

Keywords: East-West educational paradigms, elementary school, mathematics education, reciprocal learning.

Mots clés : Paradigme éducationnel Est-Ouest, école élémentaire, l'enseignement des mathématiques, l'apprentissage réciproque.

Introduction and Theoretical Background

In the past several decades, comparative studies in mathematics education have gained significant attention, giving rise to increasing interest in classroom interactions as they pertain to the teaching and learning of mathematics. Thus, general patterns that have developed in school teaching of mathematics across a variety of countries that differ geographically and culturally have been studied. Results from a number of these comparative studies in the East-West educational paradigms and arenas (Cheng, 2014; Ezeife, 2014; Moreno-Garcia, 2012; Peng & Song, 2014, amongst others) commonly draw attention to the distinctions between Eastern and Western cultures, namely, the Chinese Confucian Heritage Culture (CHC) tradition and the Greek/Latin/Christian tradition, or geographically, between East Asian countries with Confucian culture and European or English-speaking countries of European cultural backgrounds. These results further reveal that mathematics

education in Eastern and Western cultures can be characterized by sharp distinctions, such as the focus on the acquisition of basic knowledge in the East as opposed to the emphasis on creativity in the West (Kaiser & Blömeke, 2013).

Along this line of research, Watkins and Biggs (2001) investigated the learning processes of Asian students brought up in the CHC tradition and the teaching processes of Asian teachers in CHC classrooms, and identified two apparent paradoxes (Siu, 2004, p.158), viz.,

(1) The CHC learner paradox: CHC students are perceived as using low-level, rote-based strategies in a classroom environment which should not be conducive to high achievement, yet CHC students report a preference for high-level, meaning-based learning strategies and they achieve significantly better in international assessments.

(2) The CHC teacher paradox: Teachers in CHC classrooms produce a positive learning outcome under substandard conditions that Western educators would regard as most unpromising.

In his article “In search of an East Asian identity in mathematics education,” Leung (2001) described important differences between the East Asian and Western traditions in mathematics education using six dichotomies. The first dichotomy is the “product (content) versus process.” According to Leung (2001), in East Asian mathematics classrooms, mathematics content and procedures or skills are emphasized, putting basic knowledge and basic skills in the forefront, whereas Western mathematics education in the last few decades tended to focus more on the process of doing mathematics. Second, the “rote learning versus meaningful learning” dichotomy—rote learning and memorization are seen as legitimate and necessary ways of learning, contributing to a better understanding in East Asian countries. In contrast, Western cultures emphasize the necessity of understanding the phenomenon before it can be memorized and internalized. Third, the “studying hard versus pleasurable learning” dichotomy, which refers to traditional views in East Asian countries in which studying is a serious endeavour relying on hard work and perseverance. This is in contrast to many Western views, which put the child in the centre of the learning process, such that the child enjoys a meaningful learning process. Fourth, the “extrinsic versus intrinsic motivation” dichotomy in which Leung (2001) points out that on the motivational level, Western educators value intrinsic motivation in learning mathematics more than extrinsic motivation. In contrast, their Eastern counterparts emphasize the necessity of extrinsic motivation as complementary to intrinsic motivation, reflecting the high relevance of high-stake tests. The fifth dichotomy corresponds to a different understanding of the nature and the role of the teacher, which is based on social orientations in East Asian countries. Whole-class teaching with the teacher as the role model is regarded as highly important in East Asian countries, in contrast to the stronger focus on individualized learning in Western countries that lay emphasis on the independence and individualism within learning. The sixth dichotomy refers again to a different understanding of the role of the teacher, namely as a scholar with profound subject-matter knowledge in East Asian countries as opposed to the teacher as a facilitator with profound pedagogical competencies in the West.

More recently, utilizing the Learner’s Perspective Study (LPS) dataset, Leung, Park, Holton, and Clarke (2014) compared eighth grade algebra teaching across a variety of countries. In particular, a comparison was made between algebra lessons in the Confucian-Heritage Culture (CHC) countries and Western countries. Their results show that a common emphasis of algebra teaching in CHC countries is the “linkage” or “coherence” of mathematical concepts, both within an algebraic topic and between topics. On the other hand, contemporary algebra teaching in

many Western school systems places emphasis on the use of algebra in mathematical modeling in “real world” contexts and in the instructional use of metaphors where meaning construction is assisted by invoking contexts outside the domain of algebraic manipulation, with the intention of helping students to form connections between algebra and other aspects of their experience.

In all, research studies on the East-West educational paradigms have yielded valuable insights into the commonalities and differences of mathematics education at the cultural level. However, these comparative efforts employ methods of video studies or large-scale international studies, thus the data from them are “static” and lack interaction between the objects of comparison. In this study, a naturalistic inquiry-based approach was employed by the researchers to promote continuous interactions between two Canadian and Chinese elementary schools; research data were then generated naturally through the process of these interactions. It was in the supposition of the researchers that this approach would enable them to decipher more accurately and vividly the commonalities and differences in the teaching and learning engagements between the two schools. This, in turn, would lead to a better understanding of the mathematics teaching and learning approaches in the East-West educational paradigms. With this in mind, the following research question was formulated to guide the study: What are the commonalities and differences in the teaching and learning of mathematics in the two Canadian and Chinese schools? This research question was addressed by analyzing the data collected from the interactions between the two schools.

The Research Context

The Reciprocal Learning Between Canada and China Project

This study was conducted under a seven-year broader Reciprocal Learning Partnership Project in teacher education and school education between Canada and China. The project is funded through a Partnership Grant of the Social Sciences and Humanities Research Council of Canada [SSHRC] (Xu & Connelly, 2014). It involves two Canadian and five Chinese universities, two Canadian School Boards and over 40 Canadian and Chinese schools. The primary purpose of the SSHRC research is to build a knowledge base for understanding and comparing educational views on Canadian and Chinese educational systems, and for contributing to a knowledge-based public discussion of the reciprocal educational impacts of Canada and China. Researchers from the participating universities have teamed up with elementary and secondary school teachers, administrators, and district advisors to implement a long-term reciprocal learning relationship between schools in Canada and China. The project has four research teams—the mathematics education team, general education team, teacher education team, and science education team. The primary goal of the mathematics team is to provide a platform for Canadian mathematics teachers to build long-lasting and meaningful relationships with Chinese mathematics teachers so that they can learn about one another’s cultural perspectives on mathematics education and how these perspectives affect their teaching (Zhu & McDougall, 2017). The mathematics team has created two teams of researchers and sister schools in order to understand and compare the two systems in mathematics education. In this paper, the researchers report the results from one of the research teams arising from interactions with principals and mathematics teachers from a pair of second-grade classes in two sister schools.

The Canadian and Chinese Elementary Schools’ Context

For anonymity, the researchers use School A and School B to stand for the Canadian school and Chinese school respectively. School A is located in Windsor, Ontario, Canada. It is a public school

(junior Kindergarten–Grade 8) with a student population of 300. The school has two early years' classes and four primary classrooms. The school also has teachers for learning support and special education. Additionally, it has an extensive interschool program for boys and girls. Sports programs include soccer, baseball, basketball, volleyball, badminton, track, cross country, and floor hockey while school clubs include minecraft, coding, chess, reading, crochet, juggling, cake decorating, and Lego robotics. Staff and students of the school embrace technology and a variety of devices; smart boards, iPads, laptops, Apple TVs and data projectors are used extensively in the classrooms. The school focuses on character education with monthly assemblies that recognize character traits; for example, respect, caring, self-discipline, etc. Mathematics is the school's academic focus. Usually students complete school-wide problems of the week using a consistent problem-solving model, and have problem-solving strategies posted in their classrooms.

School B is located in Chongqing, China. It is a public school with 3000 students ranging from junior Kindergarten to Grade 6. As in School A (the Canadian school), School B also has an extensive interschool program for boys and girls. Interactive whiteboard and data projectors are the extensively used technological devices in the classrooms. There are evening classes, and teacher-helpers for students stay the whole week in the schools, but there are no special education teachers. The school is a prestigious primary school in Chongqing. There are currently 72 classes and 40 mathematics teachers whose main duty is to teach mathematics (normally two lessons per day for two classes).

Research Methodology

Research data were collected from direct and indirect interactions between the pair of sister schools. To facilitate communication and collaboration between the two schools, asynchronous and synchronous online interactions such as Skype, QQ, and WeChat, email communications, telephone calls as well as the Pepper software system and Blackboard system were extensively employed as research tools. This enabled the administrators and teachers to share teaching materials, create teaching videos, pose questions, and jointly prepare new teaching tasks. Also, a series of mathematics education activities including observation of lessons, mathematics education meetings, presentations, round-table group discussions spanning one to three days were undertaken. These activities, aimed at achieving a deeper understanding of the functioning and delivery of mathematics education in Canada and China (hence reciprocally learning from each other), were engaged in during the annual mutual school visits of mathematics teachers, school administrators, and other delegates from both Canada and China. In addition, the teachers and school administrators consistently engaged in detailed formal and informal discussions and interactions with the researchers through the school visits and associated teaching/learning sessions and activities.

In total there were six Skype meetings between the pair of sister schools, during which researchers and teachers interacted and discussed a number of issues in mathematics education, including the use of textbooks, aids, teaching resources and manipulatives, preparation and use of lesson notes, teaching units, lesson plans. Questions were also raised by mathematics teachers from both school related to student classroom behaviours and attitude towards learning mathematics. The teachers then shared their teaching resources after the meetings. This study recorded activities of an all-day onsite session during which a visiting team of Canadian teachers consisting of two elementary mathematics teachers, two school principals (who themselves were mathematics teachers), and one mathematics education researcher interacted extensively with their sister school counterparts in Chongqing, China; and an informal discussion with Chinese teachers about their experiences on their visiting in their sister school counterparts in Windsor, Canada. Besides, the

researchers conducted five formal discussions with the Chinese teachers and three formal discussions with the Canadian mathematics teachers. The discussions ranged in duration from one to one-and-half hours. Some of the discussions were taped, but most were not. However, the project graduate assistants took detailed notes during all the discussions and elaborated on them after each meeting/interactive session. The discussion notes were later entered into a word processor. Additionally, the researchers made five research field trips to the project school in China and four trips to the project school in Canada for practical (logistics) reasons such as initiating the project, guidance on the use of Pepper software and Blackboard system, action plans for yearly Skype meetings between the two sister schools, etc. During the trips, the researchers also engaged in many informal conversations in settings such as teachers' lounges, playgrounds, libraries, and mathematics teachers' group working rooms, etc. Although these conversations were not quoted in this study, some useful pieces of information were tapped from them. In total, 168 pages of notes were taken. Finally, the researchers also gathered documents from both schools as part of their research data; obtained mathematics textbooks used in both schools and some teaching materials from mathematics teachers and principals in both schools.

In order to understand the differences and commonalities in the teaching and learning of mathematics in the pair of sister schools, the researchers applied curricular, instructional, and learning perspectives to the data collected to seek answers. First, at the curricular front, the researchers mainly examined textbooks used in mathematics teaching, aids, resources, and manipulatives teachers use in the classroom. Next, at the instructional level, the researchers adopted Stigler and Hiebert's (1997) framework for mathematics teaching to identify the following: the kind of mathematics taught in a lesson; how mathematical concepts or procedures are presented to the students; the expectations for students during the lesson; whether the teacher used lecture directly summarized and/or selected problems that require students' thinking; and the organization of the lessons. Finally, from the mathematical learning angle, the researchers focused on student classroom behaviours and attitudes to mathematics learning. And the researchers adopted a systematic qualitative procedure—the grounded theory approach (Corbin & Strauss, 2008)—to code the data. This approach is deemed suitable because it corresponds with objective of the study which “explains an educational process of events, activities, actions, and interactions that occur over time” (Creswell, 2014, p. 432). The data were scrutinized in a few steps: first, they were summarized into relevant information on mathematics education; second, those parts of the interviews and Skype meeting notes that were in Chinese were translated into English; third, coding categories were developed to code and sort data by category; fourth, data within each category were compared to one another, and findings from different data sets were also compared. Among these steps, the third was the key step in which the original broad curricular, instructional and learning categories were replaced by more specific categories including the use of manipulatives, problem solving, parental involvement, and the issue of students with learning difficulties. These new categories emerged from the real data and reflected issues in which teachers from both schools expressed the highest degree of interest. The emergent new categories were then further subcategorized. The data analysis method (grounded theory) adopted in this study is considered appropriate and fits the circumstances of the study since the theory is “grounded” in the data; “it provides a better explanation than a theory borrowed ‘off the shelf’, and it fits the situation, [and] actually works in practice” (Creswell, 2014, p. 432). In the end, the researchers analyzed the data individually but discussed the findings as they developed a mutually acceptable interpretation of the data, and then recorded their findings.

Results and Discussion

Our results show that there are certain recurring thematic features that typify the teaching and learning of mathematics in the two schools. In general, at a global level, we see a degree of commonality across the two schools with a common emphasis on some thematic issues in the teaching and learning of mathematics including the use of manipulatives, multiple solutions to mathematical problems, and parental involvement, but also some differences between the two schools in teachers' strategies for teaching problem solving, students' learning tendencies and school supports for special needs students. However, when looking at the commonalities at a local level, the researchers still observe some degree of differences in these thematic issues between the two schools. These findings are reported in detail below.

Commonalities in the Teaching and Learning of Mathematics Between the Two Canadian and Chinese Schools

The Use of Manipulatives

In mathematics, a “manipulative” is an object which is designed in such a way that a learner can perceive some mathematical concepts by skilfully handling (manipulating) it. Current research has established a substantial relationship between the use of manipulative materials and students' achievement in the mathematics classroom, which shows that mathematical manipulatives play a key role in young children's mathematics understanding and development (Kelly, 2006). The major theoretical rationale for the use of manipulative materials has been attributed to the works of Piaget, Bruner, and Dienes who suggested that children's concepts evolve through direct interaction with the environment, and materials provide a vehicle through which this can happen (Post, 1981). The results of this study show that manipulatives are used in both schools for the purpose of facilitating students' understanding of mathematical ideas and important mathematical concepts. A typical example is that, to help students understand the underlying mathematical rationale of regrouping, in the Canadian school, a cube is widely used as a basic tool for students to count, while in the Chinese school, students use sticks to learn how to regroup and count. Figure 1 (Van de Walle, Karp, Bay-Williams, & McGarvey, 2018, p. 191) and Figure 2 show the cube and the sticks used respectively in the two schools. However, besides this commonality, the researchers find that the extent to which manipulatives are used in both schools is different. In general, in the Canadian school, manipulatives are used more frequently than in the Chinese school. According to a Canadian teacher, “students don't have a textbook or blackline masters—they spend a lot of time in groups using manipulatives,” and there are a variety of manipulatives, including “snap cubes for counting; bugs for counters; shapes, geoboards using elastics to manipulate shapes; calculators, pattern box, dice—used for games, counting and number recognition; play money, spinners to make hands-on-games to practise their knowledge; and 3D solids.”

Canadian teachers also emphasized that they hope “students learn abstract mathematics from concrete mathematics by letting them experience and operate manipulatives.” In an 11-minute video about learning fractions sent by Canadian teachers, it was shown that students spent almost 8 minutes exploring with manipulatives to help them understand concepts related to fractions, or solve fraction problems. Also, students used pattern blocks to show equal fractions, and fraction strips to figure out all the equal fractions.

Fig.1: Cube Used to Learn Regroup (Reprinted with permission by Pearson Canada Inc. *Elementary and Middle School Mathematics: Teaching Developmentally*, 5th Canadian Edition. (Van de Walle, Karp, Bay-Williams, & McGarvey, 2018, p.191. ISBN: 9780134095912)

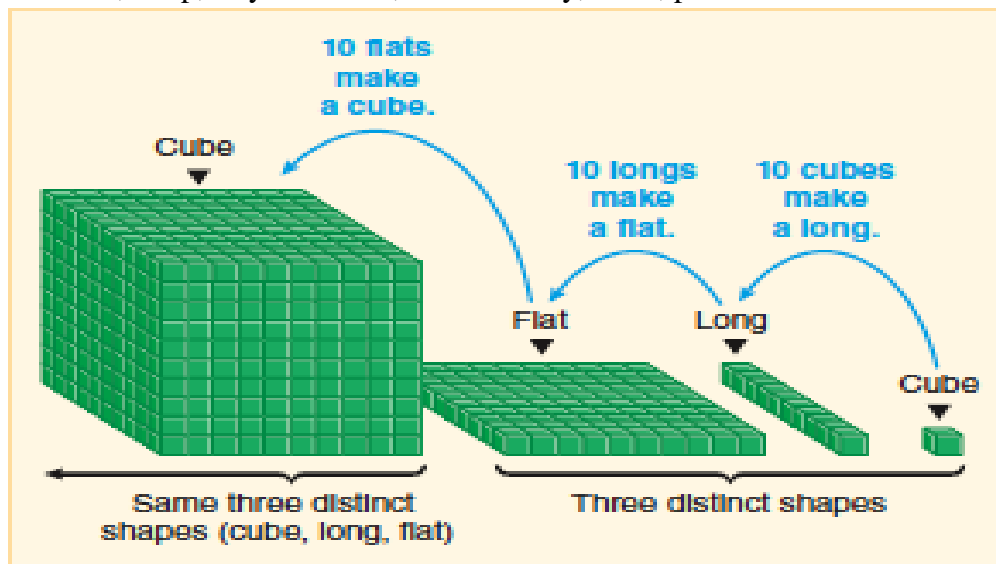
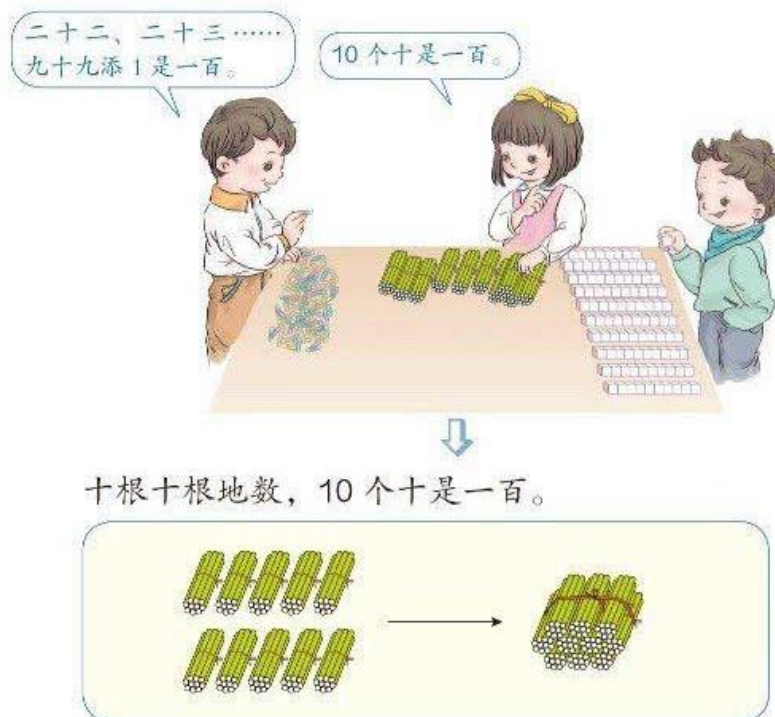


Fig. 2: Sticks Used to Learn Regroup



However, in the Chinese school, students do not use manipulatives as extensively. Cards, shapes, and sticks are often the dominant manipulatives used for mathematics learning whenever necessary during whole-class teaching sessions. In the three 40-minute mathematics lessons

observed, (a total of 120 minutes of lesson observation during the one-day onsite session of the Canadian team visit to the Chongqing sister school), manipulatives were used for only 3 minutes in one lesson while teaching the concept of division-with-remainder. During these class sessions, the students tried to show the underlying pattern of alternations of shapes by arranging two different shapes so as to understand the meaning of division. Though Chinese teachers appreciated the various inspiring manipulatives used in their Canadian counterpart school, they expressed that they used them mostly in Kindergarten. The different frequency of manipulatives usage in the two schools was not only discerned from both the Canadian and Chinese teachers' classroom observations, but was also deduced from three Skype meetings where two groups of Canadian and Chinese students presented mathematical games or problem-solving activities to each other. In these exchange sessions, Canadian students referred to and showed manipulatives when they presented mathematical tasks, whereas Chinese students presented them orally and in writing. These findings tend to suggest that the difference in the frequency of the usage of manipulative tools in the two sister schools, and the difference in Canadian and Chinese teachers' perceptions about their usage, which, in turn, reflect the teachers' beliefs about mathematics teaching and learning in general. This raises an interesting question for the researchers to investigate in a follow-up (further) study.

Multiple Methods of Solving a Problem

The importance of using different ways to solve mathematical problems is widely recognized in mathematics education. Leikin (2007) argued that “solving problems in different ways as a (meta-mathematical) habit of mind both require and foster advanced mathematical thinking” (p. 234). Commenting further on this point, Leikin and Levav-Waynberg (2008) emphasized that “solving problems in multiple ways contributes to the development of students' creativity and critical thinking” (p. 234). Schoenfeld (1983) also drew attention to the importance of solving problems in different ways and noted that when students perceive that the problem at hand can be solved or is allowed to be solved in different ways, their mathematical thinking ability improves. It is not surprising to find in this study that encouraging students to use multiple strategies to solve problems is emphasized in both schools. Research data of this study show that when given a mathematical problem, students in both schools were encouraged to use multiple methods to solve it. Figure 3 shows three different strategies that the Canadian students used when solving the mental mathematical problem: $24 + 56$ to their Chinese counterparts. The Canadian students proceeded as follows: Method 1: $24 + 50 + 6 = 80$; Method 2: $20 + 50 + 4 + 6 = 80$. Finally, the students used a third method—the widely known base-ten-blocks (manipulative tools) approach to do their computation. Thus, they counted out 7 longs (tens) = 70; and 10 cubes (ones) = 10. Hence, grouping the longs and cubes together, they arrived at $70 + 10 = 80$. In contrast, the Chinese students mainly adopted mental methods to arrive at a solution (answer) to this particular problem.

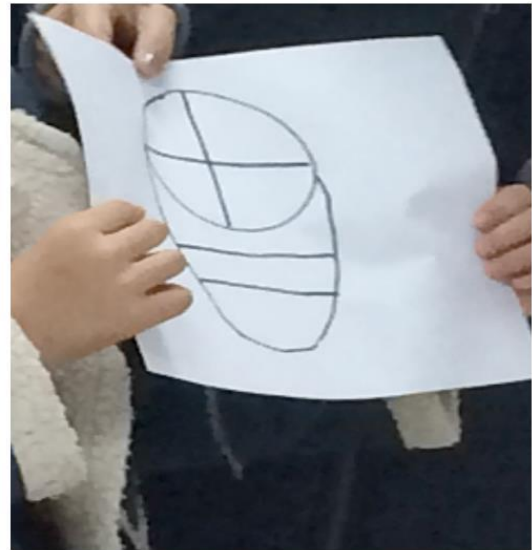
For another problem—How can you divide a mooncake into 12 pieces by cutting it four times?—the multiple-method approach to solving a mathematical problem was exemplified by the strategies adopted by the Chinese students in solving the problem to their Canadian counterparts. Figure 4 shows the visual presentation of one of the two different strategies, explained verbally as follows: First, draw a picture to present/show the mooncake. Next, use different coloured pens to show the different possible ways of cutting, since there are two methods of cutting. The first method/way is to have three horizontal cuttings and one vertical cutting, and the other method is to have one horizontal cutting and three vertical cuttings. When given the same problem, the Canadian students discussed with one another in a collaborative (group) format, and resorted to

solving the problem with manipulatives. Based on these data, we posit that though multiple methods of solving a mathematical problem are emphasized in both sister schools, students' solutions are often impacted by the methods they often use and are taught. Furthermore, regarding the evaluation of different methods, Canadian and Chinese teachers differ slightly in their perspectives. Canadian teachers mentioned that they would invite students to discuss all the methods but they would not suggest the optimal methods, instead they would allow the students to decide and use the methods they like, whereas Chinese teachers mentioned that they would discuss the advantages and disadvantages of the different methods and give suggestions on when to use the different methods.

Fig. 3: Multiple Strategies in Number Talk



Fig. 4: Multiple Strategies in Division



Parental Involvement

Much research exists about the importance of parental involvement in mathematics education and in general education. Research overwhelmingly indicates that parental involvement not only positively affects student achievement, but also contributes to higher quality education and better overall school performance. The results of this study reveal that parental involvement is a common emphasis in the two research schools in the way that both schools encourage parents to get involved in their children's mathematics education. For instance, in two Skype meetings, teachers from both schools extensively discussed the issue of parental involvement. For example, the Canadian teachers described how parents get involved in the school: "Workshops are held to help bridge parents' type of learning from their generation with the present style of mathematics education. The workshops are hosted by a program consultant, and are sponsored by the Parent Involvement Committee." The Chinese teachers also shared a few different ways of parental involvement in their school including "Communicate with parents daily; use social media—WeChat for each class and parents; students discuss with parents what they learned every day and then parents give them feedback. Also, teachers invite parents to come into the school to work on mathematics problems."

However, besides this commonality, the researchers also find that the types of parental involvement in the two schools are different. Epstein (2001) highlighted six types of parental involvement, namely, parenting, communication, volunteering, home tutoring, involvement in decision-

making, and collaboration with the community. Epstein's model presents family, school, and community as overlapping spheres of influence, the congruence of which is of considerable importance for the optimal development of children. According to Epstein's classification, the model in the Canadian school can be regarded as "collaboration with the community," where parents serve from the community to strengthen school programs. As a teacher stated:

There are "expectations" in each grade level and parents are supposed to help with these. Items that go home at night shouldn't be for evaluation since they are just for practice. We are trying to engage the parent community, because mathematics has changed from when they were younger, so they have parent and children mathematics nights.

In the Chinese school, the model adopted can be regarded as "home tutoring" where parents see their children's schoolwork as family-obligation activities, and parents play the role of teachers' partners in managing their children's mathematics learning. As a Chinese teacher put it:

The parents like to see mathematics homework examples at home so as to find out what their children are working on. The teacher talks to the parent group and shares information. Grades one and two have no homework, but the child has to have a paper signed showing that they discussed what they were working on at school with the parent, and if a child falls behind or is weak in an area, then the parent comes to school ... normally parents are literate in mathematics in China, but teachers do not expect parents to teach the mathematics content, rather parents just make sure the children do their work. Also, teachers use WeChat to talk to parents.

Differences in the Teaching and Learning of Mathematics Between the Two Canadian and Chinese Schools

Teachers' Different Strategies for Teaching Problem Solving

Problem solving has long been a staple of school mathematics (Stanic & Kilpatrick, 1988). The development of students' abilities to solve problems has remained one of the fundamental goals of school mathematics over the years, and the teaching of problem solving has always been the focus of both research and practice. The results of this study show that in the two Chinese and Canadian schools, teachers use different strategies for teaching problem solving. In the Chinese classroom, problem posing is emphasized. A Chinese teacher thinks that "the ability of problem posing is a rather important ability and requires higher thinking." In one of the Skype meetings, a Chinese teacher asked her students to prepare a mathematics newspaper themed on Chinese traditional festivals. Based on this theme, the students were encouraged to pose some relevant mathematical problems. One of the second grade students posed such a high level mathematical problem as depicted in Figure 5. The problem reads as follows: "If the mid-autumn lamps in two different shapes are decorated in such a way that one star represents one circle, next one star represents two circles, next one star represents three circles...what would be the shape of the 24th lamp"? There are both theoretical arguments (Kilpatrick, 1987), and empirical results (Cai & Hwang, 2002), which suggested that strong problem posers are also strong problem solvers. This point aligns with a recent study conducted by Cai, Moyer, Wang, Hwang, Nie, and Garber (2013) who used problem posing as a measure of the effect of middle-school curriculum on students' learning in high school. In the study, these researchers found that there was a strong connection between the students' ability to solve problems and their ability to pose valid problems within the same mathematical context. In contrast to their Chinese counterparts, this study finds that the Canadian mathematics teachers use a systematic or methodical way to teach mathematical problem solving. In the Skype meetings, the Canadian students narrated how their mathematics teacher used this strategy, called CUDDLE, as shown in Figure 6. The strategy adopts a step-by-

step procedure: **C**ircle the important facts, **U**nderline the question, **D**ecide on a strategy, **D**o the work, and then, **L**ook. Does it make sense? **E**xplain the solution. Research shows that monitoring and reflecting during problem solving helps students think about what they are doing and why they are doing it, evaluate the steps they are taking to solve the problem, and connect new concepts to what they already know (Hohn & Frey, 2002). The systematic way the Canadian teacher used is a specific strategy to assist students in monitoring and reflecting on the problem-solving process.

Fig. 5: Student's Posed Problem

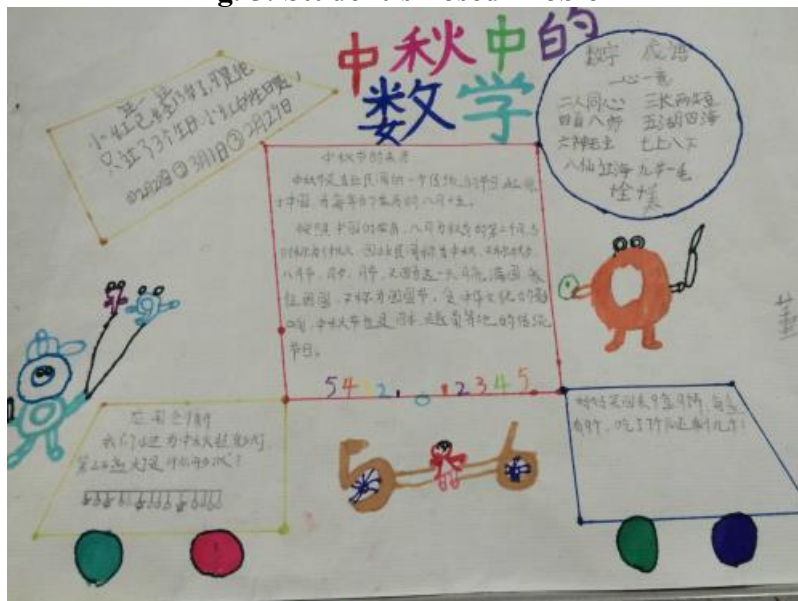
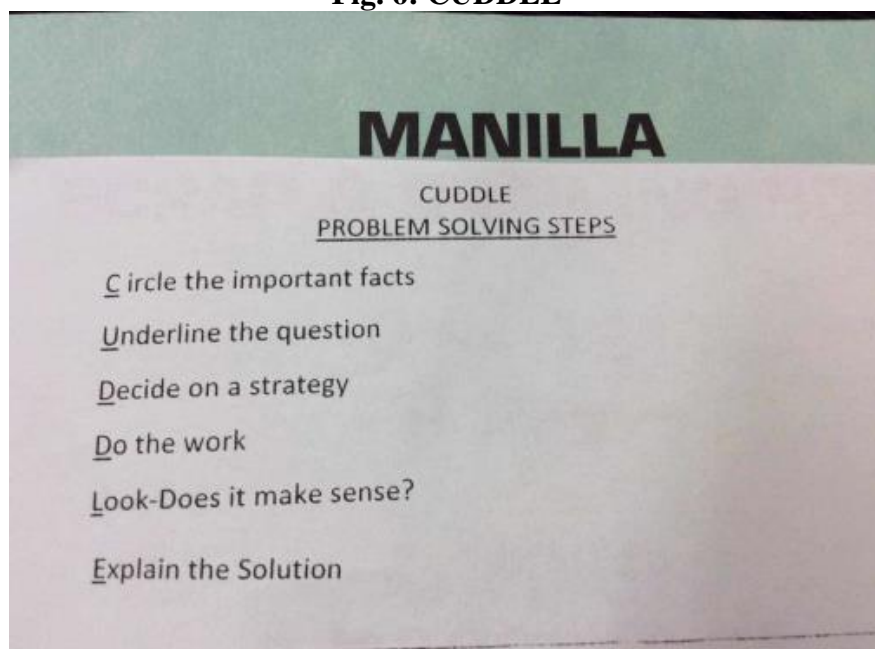


Fig. 6: CUDDLE



Students' Different Learning Tendencies

In Leung's (2001) study, he found a dichotomy in the whole-class teaching and individualized learning practices in the classroom between Eastern and Western countries, in which the latter had a stronger focus on the individual—stressing the independence and individualism within learning. However, the results of this study revealed a contrary situation since it was found that when given a mathematical problem, the Chinese students tended to solve it individually, while the Canadian students would gather together and solve it in a cooperative way. This phenomenon was also deeply reflected upon by one of the Chinese teachers:

Cooperative learning seems a natural learning habit to the Canadian students. When given a problem, they immediately gather together. This action seems very ordinary and simple, but reflects the idea that cooperative learning is deeply rooted in the students' minds.

The same observation was reiterated by the Chinese teachers after they returned from visiting their sister school in Windsor, Canada:

We were amazed by their natural and mature organization of group work. There is no “I,” only “We” for them. And we plan to do some small teaching experiments by learning from them and applying cooperative learning in our teaching.

Schools' Different Supports for Special Needs Students

How to treat students who have special needs in mathematics is an issue of concern to teachers in both schools. Nevertheless, the results of this study show that the issue is addressed in two totally different ways in the two schools. In the Canadian school, there are special education teachers who are responsible for those students, and there are various resources such as mathematics books for this group of students. Broadly in China, there are special education schools for students who have physical and developmental disabilities. However, for students who have no physical disabilities but learning difficulties in mathematics (or other subjects), there are no special supports for them. Therefore, in the Chinese school, there are no special education teachers, and normally it is the regular class teachers who assume the responsibility of helping their students to deal with the problems after school. Below is a quotation from a Chinese teacher: “When we see children struggling, we help them after school. We do not have special education...I would like to see how they (Canadian teachers) handle the problems when students have difficulties in reading or computing.” The Canadian teachers described various ways of helping with special needs students: “Use differentiated instruction for each student; perform initial testing and use psychological testing to guide the teacher where to start; oral assessments—CASI reading test for older students; one-on-one with student; small group work learning text; hands-on learning; use technology; explain mathematics terminology; use manipulatives.” The Canadian teachers also expressed surprise with regard to the accommodation of special needs learners in regular Chinese classrooms:

There are numbers of students in a class with the same learning speed, so how can the teachers manage students who have learning difficulties, especially in the situation that they learn much harder mathematics than our students, the contents of which are almost two years ahead of us?

Conclusion and Further Study

Through this interactive study on the teaching and learning of mathematics between two Canadian and Chinese schools, the researchers have come up with new knowledge on the commonalities and differences in the teaching and learning of mathematics in two schools in two countries—Canada and China—which differ culturally and geographically. Based on the findings of the

study, the researchers came to the conclusion that there are more similarities than differences in teaching and learning of mathematics in the two Canadian and Chinese schools at a global level; whereas it is the opposite at the local level. However, the situation also raises some pertinent questions, such as, why do the differences exist in the first place, and how should these observed differences be addressed? Furthermore, it is necessary to draw attention to the fact that this study has been conducted for only one and half years, thereby scratching the issue merely at the surface. So, it is the belief of the researchers that as this ongoing study further explores these differences over a longer time duration through continuing interactions among students, teachers, and researchers in the two sister schools and communities, more data could emerge, which would, hopefully, reveal more thematic features that typify the teaching and learning of mathematics in both countries. This is the future direction this ongoing cooperative study would pursue—as we explore further afield in the ever-evolving and fertile sphere of mathematics education.

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